Abstract: In this study, a rational and efficient optimal seismic design method for bridge system subjected to devastating earthquakes considering performance at ultimate state is proposed. The bridge system consists of superstructure, rubber bearings, RC piers and cast-in-place concrete pile foundation. In the proposed optimum design method, the optimum solutions for the heights of rubber bearings, cross-sectional dimensions and amount of steel reinforcements for RC piers and the detail of concrete pile foundation are determined for several allowable ductile factors of RC piers considering the constraints on the relative horizontal displacements of rubber bearings to the both bridge and transverse directions, the ductile factor of RC piers, and the constraint on the cast-in-place concrete pile foundation. From the practical design the heights of rubber bearings can take continuous values, but the other variables must be selected from discrete variable sets. Therefore, the construction cost minimization problem can be expressed as a mixed discrete-continuous problem. This problem is transformed into a convex approximation problem with the estimation formulae by using the experimental design, and the dynamic behaviors and those sensitivities are calculated analytically by using the estimation formulae without analyzing the structures. The optimum design problem is solved by a classical branch and bound method with dual algorithm. In the numerical design examples, it is emphasized that the optimum solutions can be obtained efficiently by using the experimental design. It is also demonstrated that the reductions of the heights of rubber bearings and cross-sectional dimensions of RC piers can be observed by increasing the allowable ductility factor.

Keywords: Bridge system, Optimization, Seismic design, Design of experiments, Performance at ultimate state

1. Introduction

After the Hyogoken Nanbu Earthquake in 1995, the seismic design code for highway bridges, JSHB [1], has been revised in order to ensure sufficient ultimate dynamic capacities in the bridge systems for large displacements caused by devastating earthquakes. Recently, the performance-based design method has been introduced for the seismic design at ultimate state in the JSHB. According to the JSHB, the bridge members are not allowed to yield for the frequent earthquakes (Level1), and the bridge members must have the sufficient ultimate dynamic capacities to be able to repair those rapidly after the excitations due to devastating earthquakes (Level2). This task accompanies with tremendous complexity in the process of design of the bridge system. In general, Level2 design is critical for determinations of member sizes for large scale bridge systems. Therefore, the establishment of a rational and efficient optimal seismic design method, which can determine the optimum member sizes considering performance at ultimate state in the Level2 design process, has been awaited expectantly in the practical design.

From this point of view, one of authors proposed an optimal seismic design method using the design of experiments and suboptimization technique [2,3]. In this research works, authors made effort to introduce several relations between construction cost and design variables to make the optimization problem simple. The design variables for bridge members are dealt with as continuous variables and the optimum solutions considering the displacement constraints for bridge direction are determined.

In this study, an rational and efficient optimal performance-based seismic design method for bridge system subjected to devastating earthquakes is proposed. In the design of a bridge system, the
dimensions of superstructure are assumed to be given, and the heights of rubber bearings, cross-sectional dimensions and amount of steel reinforcements for RC piers, and numbers of piles and the diameters of piles in the cast-in-place concrete pile foundation are taken into account as design variables. The dynamic nonlinear behaviors of the bridge system are analyzed precisely by using the general purpose nonlinear analysis software (TDAP-III) with the acceleration specified in the JSHB. The relative horizontal displacements between superstructure and piers to the both bridge and transverse directions are dealt with as design constraints for the rubber bearings. The ductile factor, which is given by the ratio of working curvature to the yield curvature, is dealt with as the design constraints for the RC piers so as to ensure the performance specified at the ultimate state. Furthermore, the constraint on the cast-in-place concrete pile foundation is also dealt with to ensure the sufficient ultimate dynamic capacity in the RC pile foundation. However, the constraint on the RC pile foundation is not treated in the optimization process to simplify the optimization algorithm. After determination of optimum solution the constraint on the RC pile foundation is examined, and the RC pile foundation is replaced with the larger one so as to satisfy the constraint.

From the practical design the heights of rubber bearings can take continuous values, but the other variables must be selected from discrete variable sets. Therefore, the construction cost minimization problem can be expressed as a mixed discrete-continuous problem, and it is solved by a classical branch and bound method [4] with dual algorithm and convex approximation [5,6] in this study. The sensitivities of the design constraints need in the optimization process and we encounter the difficulty to obtain those in utilizing the general purpose nonlinear analysis software. To overcome this problem the design of experiments is applied successfully in order to calculate the dynamic behaviors and those sensitivities in the optimization process. In the design of experiments, the estimation formulae for dynamic behaviors are introduced in the expression of quadratic functions of the design variables. The dynamic behaviors and those sensitivities are calculated analytically by using the estimation formulae without analyzing the structures. After the determination of optimum solution the design constraints with the estimation formulae are examined by re-analyzing the bridge system using TDAP-III. In case that the design constraints violate the allowable limits, the estimation formulae for dynamic behaviors are improved and the minimum cost design problem is re-solved. This optimization process is iterated until the relative errors between the estimated design constrains and the exact ones satisfy the allowable limits.

The proposed optimal design method is applied to a five-span continuous steel girder bridge system, and the optimal solutions at various allowable ductility factors of RC pier are compared. In the numerical results, it is demonstrated that the reductions of the heights of rubber bearings and cross-sectional dimensions can be observed by increasing the allowable ductility factor. It is also emphasized that the optimum solutions can be obtained efficiently at a few iterations of improvements of the estimation formulae for dynamic behaviors. The accuracy of the estimation formulae is excellent.

![Fig.1 Five-span continuous steel girder bridge system](image1)

![Fig.2 Front and side views of piers and RC pile foundation](image2)
within 7 percent of relative errors between the exact behaviors and estimated ones.

2. OPTIMUM DESIGN FORMULATION AND OPTIMIZATION ALGORITHM

2.1 Design Model
In this study, the five-span continuous steel girder bridge system shown in Fig.1 is considered in which the superstructure is supported by six rubber bearings, RC piers and the cast-in-place concrete pile foundation. The front and side views of a pier and RC pile foundation are described in Fig.2. The lengths of piles are 15m and five types of soil conditions in stratum are considered to calculate spring constants. The reinforcements in the cross section of piers are arranged in two layers for the bridge direction and one layer for the transverse direction, and the interval of each reinforcement are fixed at 125mm as shown in Fig.3. Following an enlargement of cross section the numbers of reinforcements increase so as to keep the intervals of reinforcements. The stiffness of a RC pier is taken into account as the trilinear rigidity reduction type model (Takeda model) shown in Fig.4. The nonlinear behaviors of the bridge system for the both bridge and transverse directions subjected to devastating earthquakes are analyzed precisely by using TDAP-III in which the Type II standard strong acceleration wave motion model at the Type II soil ground specified in the JSHB is applied. In the time-history response analysis the spring constants of rubber bearings, pile foundations and superstructure are elastic, and both the superstructure and abutment are assumed as rigid body. The piers are divided into 50 segments in order to calculate the nonlinear dynamic behaviors accurately.

2.2 Optimum Design Formulation
In the design of the bridge system, the dimension of superstructure is assumed to be given and widths of rectangular rubber bearings are assumed to be 70cm and 80cm at abutment and piers, respectively. The design variables for rubber bearings are the heights of those at abutment and piers, \( h_{B1} \) and \( h_{B2} \). For the cast-in-place concrete pile foundations the numbers of piles and diameters of piles are intensively summarized as the properties of horizontal and rotation spring constants. In this study the horizontal spring constants of RC pile foundation, \( K_h \), which can be commonly used for the time-history response analysis to the both bridge and transverse directions, are considered as the design variables. The widths to the bridge and transverse directions and the amount of steel reinforcements in a cross section, \( H_p \), \( B_p \) and \( A_s \), are taken into account as the design variables for RC piers. The bridge system shown in Fig.1 is symmetrical to the centerline and the total number of design variables is six of \( B_{h1}, B_{h2}, K_h, H_p, A_s, B_p \).

Engineers have to design the bridge system which have sufficient ultimate dynamic capacities for large displacements caused by devastating earthquakes. Therefore, the relative horizontal displacements between superstructure and piers to the both bridge and transverse directions are dealt with as the design...
constraints, \( g_{h1}, g_{h2}, g_{s1}, g_{s2}, \) for the safety of the rubber bearings. Furthermore, the ductile factors are also dealt with as the design constraints for the RC piers, \( \mu \), so as to ensure the performance specified at the ultimate state. In the design of RC pile foundation, for the case that the horizontal ultimate dynamic bearing capacity for the RC pier is not enough large for the horizontal force calculated using the specified design seismic coefficient, RC pile foundation is not allowed to yield when the equivalent loads corresponding to the horizontal ultimate dynamic bearing capacity for the RC pier are applied to the RC pile foundation. For the case that the horizontal ultimate dynamic bearing capacity for the RC pier is sufficient, RC pile foundation is allowed to yield up to the ductile factor 4.0. This constraint is quite complex to deal with in the optimization process. Furthermore, we need to consider that the design variable for RC pile foundation is dependent on the design variables for RC piers. To simplify the optimum design problem, therefore, it is assumed that the design variable for RC pile foundation is independent, and the constraint on the RC pile foundation is not dealt with in the optimization process. After the determination of optimum solution the constraint on the RC pile foundation is examined.

The total construction cost minimization problem, which is expressed as the summation of bearing construction cost, \( \text{COST}_B(B_{h1}, B_{h2}) \), pier construction cost \( \text{COST}_p(K_s) \), and pier construction cost, \( \text{COST}_p(H_p, A_s, B_p) \), can be formulated as

\[
\text{find } B_{h1}, B_{h2}, K_s, H_p, A_s, B_p \text{ which minimize } \text{COST}(B_{h1}, B_{h2}, K_s, H_p, A_s, B_p) = \text{COST}_B(B_{h1}, B_{h2}) + \text{COST}_p(K_s) + \text{COST}_p(H_p, A_s, B_p) \] (1)

subject to

\[
g_{h1} = \delta_{h1} - \delta_{a1} \leq 0 \quad (2), \quad g_{h2} = \delta_{h2} - \delta_{a2} \leq 0 \quad (3), \quad g_{s1} = \delta_{s1} - \delta_{a1} \leq 0 \quad (4)
\]
\[
g_{s2} = \delta_{s2} - \delta_{a2} \leq 0 \quad (5), \quad g_{\mu} = \mu - \mu_s \leq 0 \quad (6),
\]

where \( \delta_{a1} \) and \( \delta_{a2} \) are the allowable relative horizontal displacements of bearings at abutment and piers, which are given as the products of the heights of bearings \( B_{h1}, B_{h2} \) multiplied by 2.5. \( \mu \) is the ductile factor of a pier, which is given by the ratio of working curvature to the yield curvature for the bridge direction.

In the optimum design problem \( B_{h1} \) and \( B_{h2} \) can take continuous values, but the others must be selected from a list of discrete values. In this study, \( K_s, H_p, A_s \) and \( B_p \) are selected from the following discrete sets in which three types of pile foundations summarized in Table 1 are considered to calculate \( K_s \).

\[
K_s \in \{2212657(kN/m), 2762477, 2950210\}
\]
\[
H_p \in \{2000(mm), 2100, 2200, 2300, 2400, 2500, 2600, 2700, 2800, 2900, 3000\}
\]
\[
A_s \in \{98.6(mm^2), 286.5, 387.1, 506.7, 642.4, 794.2, 956.6, 1140\}
\]
\[
B_p \in \{3000(mm), 3500, 4000, 4500, 5000, 5500, 6000, 6500\}
\]

Therefore, the construction cost minimization problem can be expressed as a mixed discrete-continuous problem. Several types of optimization techniques have been developed, and Huang and Arora [4] investigated the efficiency and reliability of those for discrete and mixed discrete-continuous problems.
Fig. 5 Macro-flow of the proposed optimum design method

In this study the optimization problem is solved by the classical branch and bound method with dual algorithm and convex approximation [5, 6] for the reason that the approach is efficient and reliable for a mixed discrete-continuous problem without any parameters.

2.3 Optimization Algorithm

In this optimization process, in general, a number of nonlinear seismic response analyses and sensitivity analyses are necessary to determine the optimal solutions. To avoid these complexity and difficulties and make the optimum design process tremendously efficient, the design of experiments [7] is applied to introduce the estimation formulae for the dynamic behaviors. The dynamic behaviors and those sensitivities are calculated by using the estimation formulae without analyzing the structure. In the design of experiments, according to the orthogonal array table \( L_{27}(3^{13}) \) [7] given in Table 2, the three levels for all design variables are assumed and the twenty seven runs of nonlinear seismic response analyses are carried out in usage of TDAP-III for the both bridge and transverse directions, respectively.

The first six factors among thirty factors in Table 1 are assigned to the design variables, \( x_1, x_2, \ldots, x_6 = \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_6 \), respectively. Assuming that the intended variable for the \( k \)th factor is \( x_k \) and the mean value of three levels (\( \hat{x}_{ki}, i = 1, \ldots, 3 \)) for the \( k \)th factor is \( \bar{x}_k \), the general form of estimation formula is introduced in the expression of quadratic functions of the design variables given as eqs.(7)-(10).

\[
y = b_0 + \sum_{k=1}^6 b_{1k} \bar{x}_k + \sum_{k=1}^6 b_{2k} (\bar{M}_{12} - M_{12}^2 + M_{22}^2 z_k^2), \tag{7}
\]

where

\[
M_{ik} = \frac{1}{n} (\hat{x}_{i1} + \hat{x}_{i2} + \cdots + \hat{x}_{in}), \quad (k = 1, \ldots, m) \tag{8}
\]

\[
\hat{z}_{ki} = \bar{x}_k - \bar{x} \quad (i = 1, \ldots, n) \quad (k = 1, \ldots, m) \tag{9}
\]

\[
z_k = x_k - \bar{x} \tag{10}
\]

\( m \) and \( n \) are respectively the number of factors, i.e. the number of design variables (= 6), and the number of levels for each factor (= 3). The estimation values of \( b_0, b_{1k}, \) and \( b_{2k} \) in eq.(7) are given as

\[
b_0 = \frac{1}{rS_k} \sum_{i=1}^r T_{ki}, \quad b_{1k} = \frac{1}{rS_k} \sum_{i=1}^r W_k T_{ki} \quad (l = 1, 2) \tag{11}
\]

where

\[
S_k = \sum_{i=1}^n W_k^2 \tag{12}
\]

\( r \) is the number of runs with the level \( \hat{x}_{ki} (= 9) \). \( T_{ki} \) is the summation of results by the design of experiments with the level of \( \hat{x}_{ki} \). \( W_k = \hat{z}_k \) is the value of function of coefficient \( f_k(x_k) \) in eq.(7) with respect to \( b_{1k} \) and \( b_{2k} \).
After the determination of optimum solutions the design constraints with the estimation formulae are examined by re-analyzing the bridge system. In case that the design constraints violate the allowable limit, the three levels for all design variables and estimation formulae for dynamic behaviors are improved and the minimum cost design problem is re-solved. This optimization process is iterated until the relative errors between the estimated design constrains and the exact ones satisfy the allowable limit.

After the determination of optimum solution the constraint on the RC pile foundation is examined. In the case that the constraint is violated the RC pile foundation is replaced with the larger one and the bridge system is re-optimized.

The macro-flow of the proposed optimization algorithm is depicted in Fig. 5.

### Table 3 Improvements of three levels in the optimization process

<table>
<thead>
<tr>
<th>Levels</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>16.0(15313)</td>
<td>14.0(22857)</td>
</tr>
<tr>
<td></td>
<td>14.0(26667)</td>
<td>12.0(32000)</td>
</tr>
<tr>
<td></td>
<td>12.0(32000)</td>
<td>10.0(32000)</td>
</tr>
</tbody>
</table>

### Table 4 Optimum solutions for $\mu = 2.0$, 3.0 and 4.0

<table>
<thead>
<tr>
<th>Allowable ductile factors $\mu_a$</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{h1}$ (spring constant)</td>
<td>14.18cm (17273kN/m)</td>
<td>8.74cm (28035kN/m)</td>
<td>8.37cm (29257kN/m)</td>
</tr>
<tr>
<td>$B_{h2}$ (spring constant)</td>
<td>11.28cm (28366kN/m)</td>
<td>8.0cm (40000kN/m)</td>
<td>8.0cm (40000kN/m)</td>
</tr>
<tr>
<td>$K_h$ ($\phi$, $n$)</td>
<td>2762476 ($\phi=1.2$, $n=9$)</td>
<td>2762476 ($\phi=1.2$, $n=9$)</td>
<td>2212657 ($\phi=1.0$, $n=9$)</td>
</tr>
<tr>
<td>$B_r$</td>
<td>2700mm</td>
<td>2800mm</td>
<td>2700mm</td>
</tr>
<tr>
<td>$A_s$</td>
<td>1140mm$^2$</td>
<td>387.1mm$^2$</td>
<td>387.1mm$^2$</td>
</tr>
<tr>
<td>$H_r$</td>
<td>4000mm</td>
<td>4500mm</td>
<td>3000mm</td>
</tr>
<tr>
<td>$\delta_{s1}/\delta_{a1}$</td>
<td>D.exp.* 1.000</td>
<td>D.exp.* 1.000</td>
<td>D.exp.* 0.992</td>
</tr>
<tr>
<td>Anal** 0.971</td>
<td>Anal** 1.011</td>
<td>Anal** 1.013</td>
<td></td>
</tr>
<tr>
<td>$\delta_{s2}/\delta_{a2}$</td>
<td>D.exp.* 0.912</td>
<td>D.exp.* 0.604</td>
<td>D.exp.* 0.413</td>
</tr>
<tr>
<td>Anal** 0.895</td>
<td>Anal** 0.602</td>
<td>Anal** 0.390</td>
<td></td>
</tr>
<tr>
<td>$\delta_{s1}/\delta_{a1}$</td>
<td>D.exp.* 0.903</td>
<td>D.exp.* 0.976</td>
<td>D.exp.* 1.000</td>
</tr>
<tr>
<td>Anal** 0.967</td>
<td>Anal** 1.021</td>
<td>Anal** 0.935</td>
<td></td>
</tr>
<tr>
<td>$\delta_{s2}/\delta_{a2}$</td>
<td>D.exp.* 0.837</td>
<td>D.exp.* 0.994</td>
<td>D.exp.* 0.897</td>
</tr>
<tr>
<td>Anal** 0.847</td>
<td>Anal** 1.038</td>
<td>Anal** 0.832</td>
<td></td>
</tr>
<tr>
<td>$\mu/\mu_a$</td>
<td>D.exp.* 1.000</td>
<td>D.exp.* 0.987</td>
<td>D.exp.* 0.971</td>
</tr>
<tr>
<td>Anal** 1.012</td>
<td>Anal** 1.000</td>
<td>Anal** 1.008</td>
<td></td>
</tr>
<tr>
<td>yield of pile foundation</td>
<td>Bridge dir: $\mu_a=1.51$, Transverse dir: $\mu_a=1.62$</td>
<td>Bridge dir: not yield, Transverse dir: $\mu_a=1.62$</td>
<td>not yield</td>
</tr>
<tr>
<td>Total cost ($10^3$ yen)</td>
<td>193636</td>
<td>158974</td>
<td>138232</td>
</tr>
</tbody>
</table>

D.exp.*: Feasibility of design constraints with the estimation formulae by the design of experiments
Anal**: Feasibility of design constraints using exact behaviors by analysis
3. DESIGN EXAMPLES

The proposed optimal design method is applied to the five-span continuous steel girder bridge system shown in Fig.1 and the optimal solutions for several allowable ductile factors $\mu_a$ are compared. The unit cost of rubber is as 45yen/cm$^3$. The construction costs of a pile are assumed as 65200yen/m$^3$ for the diameter 1.0m and 73800yen/m$^3$ for the diameter 1.2m. The construction costs of footing and form for pile foundation are assumed as 33500yen/m$^3$ and 8000yen/m$^2$. The construction costs of concrete, form and reinforcement for piers are assumed as 18500yen/m$^3$, 8000yen/m$^2$ and 12000yen/ft, respectively. Following the flow-chart in Fig.5 the optimization processes for $\mu_a = 2.0$, 3.0 and 4.0 are initiated with the levels of iteration 1 shown in Table 3. In the optimization process, the lower and upper limits for discrete design variables are set at the adjacent discrete values of the minimum and maximum values of the three levels to limit improvements of design variables. The optimum solution for $\mu_a = 2.0$ can be obtained quite efficiently without any improvements of the three levels for all design variables. The optimum solutions for $\mu_a = 3.0$ and 4.0 determined by the lower limits set as the move limits. After then, the three levels are improved to the values of iteration 2 in Table 3 referring to the optimum solutions with the previous three levels. The optimum solutions for $\mu_a = 3.0$ and 4.0 can be obtained efficiently at this stage without additional improvements of the three levels. The optimum solutions for $\mu_a = 2.0$, 3.0 and 4.0 are summarized in Table 4.

The horizontal spring constants of pile foundations for all cases are determined by the lower limit which indicates the lowest cost. Then, the RC pile foundations for $\mu_a = 2.0$ and 3.0 are replaced with the larger one so as to satisfy the constraint on the RC pile foundation. In case of $\mu_a = 2.0$ the largest dimensions of cross section, $H_p$ and $B_p$, and reinforcement in the piers $A_s$ are required in order to satisfy the allowable ductile factor. By increasing the heights of rubber bearings $B_{h1}$ and $B_{h2}$, namely reducing the values of spring constant, the period of bridge system is made longer and the effect from superstructure is minimized. As the result the total construction cost is minimized. In case of $\mu_a = 3.0$ $B_{h1}, B_{h2}$ and $A_s$ are remarkably reduced compared with those in case of $\mu_a = 2.0$, and $B_{h2}$ and $A_s$ are determined by the lower limits. The total cost is reduced to 82 percent of that in case of $\mu_a = 2.0$. In case of $\mu_a = 4.0$ $B_{h1}, K_s, A_s$ and $H_p$ are determined by the lower limits. The total cost is reduced to 71 percent of that in case of $\mu_a = 2.0$.

As clearly seen from the values of feasibility of design constraints using exact behaviors by analysis in Table4, both the constraints on relative horizontal displacements at abutment to the bridge direction $g_{h1}$ and ductile factors $g_{\mu}$ are active for all cases simultaneously. The displacements at abutment and piers to the transverse direction $g_{c2}$ are also active for $\mu_a = 3.0$. The displacements at piers to the bridge direction $g_{h2}$ are inactive for all cases. The accuracy of the estimation formulae is excellent within 7 percent of relative errors. The exact constraints are enough feasible within 3.8 percent of violation for all cases.

4. CONCLUSIONS

The following conclusions can be drawn from this study:

1) The proposed optimal design method can determine the heights of rubber bearings, cross-sectional dimensions and amount of steel reinforcements for RC piers, and numbers and diameters of piles rigorously and efficiently.
2) By applying the design of experiments, the estimation formulae for the ductile factor in piers and the maximum horizontal displacements to the bridge and transverse directions can be introduced accurately with small number of nonlinear seismic response analyses. The accuracy of the estimation formulae is excellent within 7 percent of relative errors between the exact behaviors and estimated ones.

3) A few iterations of improvements for three levels are required to obtain the optimum solutions in the proposed design method.

4) In the case that the allowable ductile factor is set at a small value, the heights of rubber bearings increase in order to make the period of bridge system longer, and the effect from superstructure is minimized. As increasing the value of allowable ductile factor the heights of rubber bearings are reduced and the dimension of cross section and reinforcement in the piers are also reduced.

5) In the proposed design process, the constraint on the RC pile foundation is not dealt with in the optimization process and, then, the RC pile foundation is replaced with the larger one so as to satisfy the constraint on the RC pile foundation. This design process can simplify the optimization algorithm greatly.

6) The constraints on relative horizontal displacements at abutment to the bridge direction \( g_{ai} \) and ductile factors \( g_{p} \) are active at the optimum solutions simultaneously.

Acknowledgements
Part of this work has been supported by FUT Research Promotion Fund.

References