APPLICATION OF GA AND MATHEMATICAL MODELS IN LONG SEA OUTFALL DESIGNING AND CONCENTRATION PREDICTIONS

Lankasena B.N.S., Prof. (Mrs.) Ratnayake N., Dr. Indralingam M.
Department of Civil Engineering, Department of Mathematics
APPLICATION OF GA AND MATHEMATICAL MODELS IN LONG SEA OUTFALL DESIGNING AND CONCENTRATION PREDICTIONS

Lankasena B.N.S., Prof. (Mrs.) Ratnayake N., Dr. Indralingam M.  
Department of Civil Engineering, Department of Mathematics

ABSTRACT

Long sea out-fall is an acceptable method of domestic wastewater disposal, provided that the design is done properly, and it is constructed robustly and according to the design. The dispersion of the sewage after discharging to the sea has come under increased scrutiny in recent years, due to the sea and seashore pollution. In a properly designed and constructed long sea out-fall, the waste is discharged at a point in sea, away from any environmentally sensitive areas such as coral reefs, and far enough from the beach so that the quality of water reaching the beach conforms with stipulated standards. In this paper the development and applications of SOS (Sea Outfall System), a model developed at University of Moratuwa, will be demonstrated. SOS can be used in sea outfall designing, data analyzing and concentration predicting. The system is developed using GA optimizations and mathematical models.

Notation and Sign Convention

- SOS: Sea Outfall System
- ρ: Ambient Water Density
- Δρ₀: Discharge Density Difference
- b₁: Plume Width
- k: Decay Constant
- C₃: Discharge Concentration
- DNA: Deoxyribo Nucleic Acid
- q: Probability of selecting the best individual
- P: Population size
- Cₚₙ: Unit pretreatment cost
- aᵢ: Lower bound of GA variable i
- Dᵢ: Dispersion factor
- dᵢ: Vertical Density Gradient
- g: Gravitational Force
- GA: Genetic Algorithm
- hᵢ: Thickness of the Plume
- L: Diffuser Length
- Q₀: Discharge Flow-rate
- U: Ambient Current Velocity
- r: Rank of the individual, where 1 is the best.
- Cₙ: Unit diffuser cost
- Tᵢ: Required pretreatment level
- bᵢ: Upper bound of GA variable i

INTRODUCTION

The wastewater is discharged through diffuser jets at the end of pipeline, and rises through the water column in the form of a plume. The top of the plume may either penetrate the water column and appear on the surface, or be submerged, depending on the density differences and flow conditions. As it rises through the water column, it undergoes dilution, and dispersion in the body of the seawater. In addition non-conservative materials undergo decay with time. Therefore, the concentration of contaminants greatly reduces before the water reaches the shoreline. The length and positioning of the outfalls should be designed, so that the, concentration of critical parameters in the water reaching the shoreline are within the standards stipulated for the objective water quality of the coastal water.
Treatment of the wastewater up to a point will reduce the initial concentration of the contaminants, and therefore reduce the required length of the outfall. On the other hand, treatment will be an additional cost component. Determination of the right degree of treatment and the right length of pipe will need an optimization study.

METHODOLOGY

A mathematical model was used to model the reduction in concentration of conservative and non-conservative substances. Subsequently a user friendly computerized system (SOS) was developed for the prediction of parameters such as final concentration of contaminants at the shoreline for a given outfall length and location. Finally GA was used for optimization of the design of a sea outfall for varying waste characteristics, treatment options and environmental conditions. The SOS is developed according to Object Oriented Techniques. Major programming is done using Visual C++ Version 6.0 and MATLAB 5.3 was used in GA optimizations.

THEORETICAL BACKGROUND

The reduction in concentration is due to the processes of initial dilution, dispersion, and decay. The dilution mechanisms occurring in the near and far fields are significantly different, therefore these regions were considered separately in the analysis.

Near-field Modeling

Effluent dilution in the initial mixing region has been the subject of studies in the laboratory, in the field, and with mathematical models. Because of the combined effects of the effluent buoyancy, ambient stratification, and current, the prediction of initial dilution can be involved (Metcalf and Eddy, 1991).

Vertical Single Port Discharges

Average Plume Dilution at the end of initial mixing region for stagnant ambient and flowing ambient flow conditions can be calculated with the following equations based on dimensional analysis and laboratory measurements (Muellenhoff et al, 1985)

Average Initial Dilution for stagnant ambient \( I_{avg} = 0.13 \rho_{D}^{1/3} Q_{D}^{-2/3} H^{5/3} \) \hspace{1cm} (1)

Average Initial Dilution for flowing ambient \( I_{avg} = 0.29(U/Q_{b})H^{2} \) \hspace{1cm} (2)

These results are for the average plume dilution, which is approximately 1.8 times the minimum dilution, at the plume centerline. When the plume hits the surface, it spreads horizontally and additional dilution, by a factor of up to five occurs (Wright et al, 1990). For horizontally discharged effluent, dilutions on the order of 20 to 50 percent greater are achieved at the water surface than for vertical discharges (Lee et al, 1987). When the sea is stratified linearly,
equilibrium height of the plume depends on the vertical density gradient. Using following equations equilibrium height of stagnated and flowing plume can be calculated.

Equilibrium Height (Stagnated plume) \( H_{eqn} = 2.918 \left( \frac{g'}{g} \right)^{1/4} Q_{o}^{3/4} N^{3/4} \) (3)

Equilibrium Height (Flowing plume) \( H_{eqn} = 1.85 \left( \frac{Q_{o} g'}{N^2 U} \right)^{1/3} \) (4)

Discharge Buoyancy \( g' = (\Delta \rho_{o} / \rho) \) (5)

Buoyancy Frequency \( N = \left( -\frac{g}{\rho} \frac{d \rho}{dz} \right)^{1/2} \) (6)

Then to get the average initial dilution, depth (H) of the equations (1) and (2) should be replaced with the equilibrium height. Thus Equilibrium height of the plume and average initial dilution for the stratified sea may be written as (Metcalf and Eddy, 1991);

Average Initial Dilution (stagnated plume) \( I_{avg} = 0.138 \left( \frac{g'}{g} \right)^{1/3} Q_{o}^{2/3} H_{eqn}^{5/3} \) (7)

Average Initial Dilution (flowing ambient) \( I_{avg} = 0.29(U / Q_{o}) H_{eqn}^{2} \) (8)

Initial Dilution of Multi-port Diffuser Discharges in Linearly Stratified Sea

When compared with single port discharges, the initial cost of designing and construction of multi-port diffuser discharges is high. However, by discharging sewage into a much larger area, these would achieve a much higher dilution. Thus, in most cases, it is more advantageous to provide multi-port diffusers than single port discharge outfalls.

According to Figure 1 the lowest initial dilution is obtained when the Froude number is lower than 0.1 (Roberts et al, 1989). When the current speed is low, Froude number is also low. For low current speed the dilution is independent of the current speed and the direction. With the Figure 1 it is clear that by laying the diffuser perpendicular to the current we can achieve the highest initial dilution. According to Figure 2, maximum equilibrium plume height is obtained, when the Froude number is lower than 0.1 (Roberts et al, 1989). For aesthetic reasons it is always desirable to have a submerged plume. For this type of plume, the average dilution, \( S_{a} \) is approximately equal to the minimum dilution, \( S_{m} \) multiplied by 2.0 (Metcalf and Eddy, 1991). According to Figure 2, when the Froude Number is lower than 0.1 minimum near fields dilution can be obtained.
Figure 1: Minimum initial dilution variation with the Froude number

Figure 2: Equilibrium plume height variation with the Froude number
Minimum centerline initial dilution and equilibrium height of the plume can be calculated using following equations (Metcalf and Eddy, 1991):

Minimum Centerline Initial Dilution \[ I_{\text{min}} = \frac{g'_{D}^{1/3} L^{1/3}}{Q_{D}^{1/3} N} \] (9)

Equilibrium Height of the Plume \[ H_{eq} = 2.8 \frac{(g'_{D} Q_{D} L)}{N}^{1/3} \] (10)

Froude Number \[ F = \frac{U^{3} L}{g'_{D} Q_{D}} \] (11)

Far-field Modeling

After initial mixing, the plume is carried away by the ambient current. As a result of this turbulent diffusion, sewage undergo further mixing. The region between near field and far-field is known as the transition region. The flow-rate in the transition region is given by (Metcalf and Eddy, 1991):

Flow rate in the Transition Region \[ Q_{\text{trans}} = L_{w} \rho Q_{D} \] (12)

Maximum centerline concentration of constituent \[ C_{\text{max}} = C_{D} \rho I_{\text{min}} \] (13)

If the plume is moving at the same speed as the ambient current \[ Q_{\text{trans}} = U b_{f} h_{f} \] (14)

Plume width of diffuser discharge is very close to the length of the diffuser \( b_{f}=l \). Therefore using equation (14) the plume thickness can be determined. After releasing sewage decaying of non-conservatives take place with the time. Therefore the concentration of non-conservatives in the far-field depend on the diffusion and decaying. This can be obtained by simulating the discharge as a continuous vertical source of width, \( b_{f} \), and height, \( h_{f} \) (Metcalf and Eddy, 1991). Neglecting vertical diffusion can be justified when the ambient water is stratified or when the plume occupies the full water depth. For this case and when the far-field current is uniform, the centerline concentration and plume width are given by the following expressions (Brooks, 1960):

Centerline concentration \[ C_{w} = C_{D} e^{-x/3} \text{erf} \left( \frac{3/2}{\left[ 1 + \left( \frac{8 E_{x} x}{U b_{f}^{2}} \right)^{3/2} \right]} \right) \] (15)

Plume width at distance \( x \) \[ b_{w} = h_{f} \left( 1 + \frac{8 E_{x} x}{U b_{f}^{2}} \right)^{3/2} \] (16)

Initial Transverse Diffusion Coefficient \( b_{f} \) in feet \[ E_{x} = 0.001 b_{f}^{4/3} \] (17)

Error Function \[ \text{erf}(x) = 1 - \frac{1}{(1 + a_{1} x + a_{2} x^{2} + a_{3} x^{3} + a_{4} x^{4})^{4}} \] (18)

\( a_{1} = 0.278393, \ a_{2} = 0.230389, \ a_{3} = 0.000972, \ a_{4} = 0.078108 \)
Pretreatment of Sewage before Discharging

In Sri Lanka with the existing long sea outfalls, sewage is released to the sea with no pretreatment. Sea outfalls can be designed in such a way as to avoid pretreatment because the operation and maintenance of such an outfall is very simple. When the cost of such an outfall is affordable, it may be preferable to avoid pretreatment. Biological degradation that occurs in a treatment plant can also occur in the marine environment, which has no maintenance cost and land requirement.

However treatment may be essential for virus removal since recent studies by Katzenelson and Shuval indicate that $t_{90}$ values of viral bodies are of the order of 48 hr compared with about 2 to 4 hr for coliforms (Coastal Pollution Control, 1976), which requires very long outfall lengths to achieve the necessary virus removal. When the concentrations of pollutants are higher than the required water quality objectives, pretreatment will be essential to meet required water quality objectives.

GA as an Optimization Technique

GA is a major topic in neural and evolutionary computing. When using GA, the concept of the natural phenomenon, evolution, is used. All living organisms are made up with cells and these cells contain many organelles. Chromosome is such an organelle (strings of DNA), which serves as a blueprint for the organism, and is located in the nucleus of the cells. Each chromosome conceptually can be divided into genes. Different DNA's and the sequence of DNA are responsible for making different genes. Individuals within the same population show different adaptations and mechanisms for survival, mainly due to genetic variations. Due to sexual reproduction of parents, offspring inherits properties from their parents. Crossover is the process, which provides a mix of both parents' genes to the offspring. Sometimes mutation may occur at this stage and as a result, offspring will have abnormal characteristics when compared with other individuals of the population. This mutated trait may fit to the environment more than the normal trait or may be weaker. If the mutated trait is fit to the environment it will survive in the environment or else it will quit from the environment. A naturally selected fit individual will produce new individuals by reproduction. As generations go on traits and individuals that fit to the environment will be sustained.

Evolution process of GA for Long Sea Outfall Optimization

1. SOS provides maximum and minimum ranges for the diffuser length/outfall pipe. Pretreatment levels and cost parameters, needs to be provided by the user. Genes that make up each chromosome (individual) are lengths of the diffuser and length of the outfall pipe (Figure 3).

2. GA randomly generates an initial population. This is being done for the range, that the user required for the diffuser/outfall pipe and user should specify the number of individuals for the population.
3. GA selects individuals for reproduction and there are several mechanisms for selection in GA. An individual in the population can be selected more than once. All individuals in the population have a chance of being selected to reproduce into the next generation.

Selection of individuals is done according to normalized geometric selection [Joines and Houck 1994], and this requires only the evaluation function to map the solutions to a partially ordered set. According to this method, probability of selecting the \( j \)th individual, \( P_j \), is defined by:

\[
P_j = q'(1-q')^{j-1} \quad (19)
\]

\[
q' = q/(1-(1-q)^p) \quad (20)
\]

4. GA operators such as crossover and mutation operate at this stage to create new solutions (individual/chromosome) based on the existing solutions in the population. Different crossover and mutation mechanisms used, is given below.

**Simple Crossover**

Simple cross over generates a random number \( r \) from a uniform distribution from 1 to \( m \) and creates two new individuals (\( X' \) and \( Y' \)) according to following equations.

\[
x'_i = \begin{cases} 
  x_i, & \text{if } i < r \\
  y_i, & \text{otherwise}
\end{cases} \quad (21)
\]

\[
y'_i = \begin{cases} 
  y_i, & \text{if } i < r \\
  x_i, & \text{otherwise}
\end{cases} \quad (22)
\]

**Arithmetic Crossover**

Arithmetic cross over produces two complimentary linear combinations of the parents, where \( r = U(0, 1) \);

\[
X' = r X + (1-r)Y \quad (23)
\]

\[
Y' = (1-r)X + rY \quad (24)
\]

**Uniform mutation**

Uniform mutation randomly selects one variable, \( j \), and sets it equal to an uniform random number \( U(a_i, b_i) \):

\[
x'_i = \begin{cases} 
  U(a_i, b_i), & \text{if } i = j \\
  x_i, & \text{otherwise}
\end{cases} \quad (25)
\]
Boundary Mutation

Boundary mutation randomly selects one variable, $j$, and sets it equal to either its lower or upper bound, where $r = U(0,1)$;

$$x'_i = \begin{cases} a_i & \text{if } i = j, \ r < 0.5 \\ b_i & \text{if } i = j, \ r \geq 0.5 \\ x_i & \text{otherwise} \end{cases}$$  \hspace{1cm} (26)

Non-uniform mutation

Non-uniform mutation randomly selects one variable, $j$, and sets it equal to either its lower or upper bound, where $r = U(0,1)$

5. Whether the individual will survive and continue on to the next generation is determined by the evaluation function and evaluation is done assuming different system scenarios.

When the outfall length is constant or short diffuser length should be considered to minimize the cost and pretreatment may be necessary to acquire required water quality objectives. Initial dilution is generated by the program using the equation (9) and, total cost, $C_{tot}$ for varying diffuser length can be calculated using the following equation. For this scenario, GA is used following equation for the evaluation.

$$C_{tot} = (L \cdot C_d) + ((D_{tot} / I_{mid}) \cdot C_{pt})$$  \hspace{1cm} (27)

When the outfall length is long, this will have a significant impact on the total cost. On the other hand when the distance to the shore is high, as a result of dispersion, pollutant concentration gets reduced. Therefore pretreatment for conservatives is required only when the initial dilution and dilution due to dispersion has not met the required water quality objectives. For this scenario, GA is used following equation for the evaluation.

$$C_{tot} = (L \cdot C_d) + ((D_{tot} / (I_{min} \cdot D)_d) \cdot C_{pt})$$  \hspace{1cm} (28)

$$D_2 = C_d / I_{min}$$  \hspace{1cm} (29)

6. GA moves from generation to generation selecting and reproducing parents until it meets the maximum generation number.
Figure 3: Integrated System Architecture
INPUTS AND OUTPUTS (INTERFACES) OF SOS

1. Long Sea Outfall (Initial Mixing Data) menu (Figure 6) is for SOS to gain basic data about long sea outfall. Based on this menu, user will go to the next screens and SOS produces outputs to meet user's requirements.
2. Single Port Outfall (Initial Mixing Region) menu (Figure 7) provides initial mixing region data for different scenarios.
3. Initial Mixing Region - Multi-Port Diffuser - (Lineally Stratified Sea) menu (Figure 8) is to provide data about initial mixing region of diffuser discharges to the user.
4. Dispersion Overview menu (Figure 9) provides data to identify concentrations of pollutants due to dispersion and decaying.

5. Graphical Representation menu (Figure 10) analyzes and displays the way that pollutant disperses in the near field and far field can be identified.

6. Outfall Designing Configurations menu (Figure 11) is the interface which use to gain user required configuration for designing.

7. Decision Supporting Analysis menu (Figure 12) provides the designing configuration that the user required, and also provides decision supporting information that the user needs to be evaluated.

8. Sea Outfall Help menu (Figure 13) provides help about the system, technical terms, and other useful information on long sea outfalls.

9. GA Optimum Value achieved graph (Figure 14) is a GA output, and shows the stability of the optimum value achieved over generations.

ANALYSIS & DISCUSSION OF RESULTS

SOS is specially developed to design and identify pollutant behavior of conservatives and non-conservatives, discharged using multi-port diffusers into linearly stratified seas. The system can use with multi-port diffuser outfalls to predict concentration of parameters in the far field as well as the near field, and also can be used as a decision support system for designing. It is also capable of analyzing the near field of single port sea outfalls and can handle scenarios such as stratified and unstratified sea condition and stagnated and flowing plumes. SOS provides Wizard based menus, so the user can find their requirements with the given path. Interactive dialogues provide assistance whenever user makes configuration errors. SOS Expert (Decision-maker) provides decisions on required designing configurations, so that the user can find optimum configurations. SOS HELP provides necessary information about long sea outfalls as well as information about the SOS.

GA generates a population within the user specified boundaries. These boundaries can be selected with SOS, or user can enter randomly to suit available budget and other requirements. When the user has limited funds to invest on making an outfall, different combinations of long sea outfall length and pretreatment level can be identified using GA optimization. The system provides an easy way to determine concentrations due to dispersion and decaying. With the help of SOS Expert, designers can optimize outfall configurations, for achieving the required objective water quality.

CONCLUSIONS & RECOMMENDATIONS

Designing, construction and maintenance of sea outfalls requires a large amount of money, time and human involvement. The cost and time, which need in sea outfall designing and operations, can be reduced with SOS. Artificial intelligence feature has been associated with SOS Expert and this provides decision supportive analysis to the user. This can be used as software of demonstrating and educating about long sea outfall.

SOS satisfies user requirements and dispersion of pollutants can identify easily and also provides necessary guidance for designing. Integration of SOS with
GA optimization using MATLAB enhances the usability of SOS. SOS can be further improved to handle dynamic ambient sea situations. SOS can be further developed to handle all plume scenarios of single port discharges and multi-port diffusers. SOS already used in designing and identifying initial dilution and dispersion of outfall discharges in industrial applications.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support and assistance given by Lanka Hydraulic Institute, WS Atkins international Ltd, Colombo Municipal Council, and National Water Supply and Drainage Board. Authors also wish to acknowledge with thanks the support given by Dr M. Jayaweera, Dr. (Mrs.) H. Karunaratne and Prof. S. Hettiarachchi. Financial assistance extended by Asian Development Bank also gratefully acknowledged.

REFERENCES

“Colombo Outfall Trace Survey”, WS Atkins international Ltd, Greater Colombo Sewerage Project.
Figure 5: About us and contact information

Figure 6: Initial mixing data input interfaces
Figure 7: Initial mixing data analysis of single port discharges

Figure 8: Initial mixing data analysis of multiport discharges
Figure 9: Dispersion Overview

Figure 10: Graphical representation menu
Figure 11: Outfall designing configuring interface

Figure 12: Decision supporting data analysis menu
Figure 13: SOS Help menu

Figure 14: GA Optimum value achieved graph